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A model comparison for daylength as a function of latitude and day of year

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Abstract

A model that calculates the length of the day for a flat surface for a given latitude and day of the year is described. Calculated daylengths are within 1 minute of values published in Smithsonian Meteorological Tables and the Astronomical Almanac for latitudes between 40° North and South with a maximum error of 7 minutes occurring at 60° latitude. The model allows the use of different definitions of sunrise/sunset and the incorporation of twilight. Comparisons with other daylength models indicate that this model is more accurate and that variation in accumulated hours of daylight of up to one week over the course of the growing season can be accounted for by how sunrise/sunset are defined.

Keywords: Daylength; Model comparison

1. Introduction

Many ecological and agronomic models require knowledge of daylength. Ritchie (1991), for example, uses daylength to model physiological/ontogentic phenomena in the CERES wheat model. In FOREST-BGC, Running and Coughlan (1988) use the latitude of the modeled location and the day of the year as input to a daylength submodel used in calculations of canopy transpiration, photosynthesis, and energy balance. Nikolov (1992) uses Running and Coughlan's daylength equation set to model solar radiation. In addition, if one desired to use Campbell's radiation model (1977) for daily radiation, a daylength model could be used to determine the range over which to integrate.

Another need for daylength is to calculate accumulated irradiance. An example of a model using this technique is the soybean model, Soyphen, (Hodges and French, 1985) which uses daylength and temperature to estimate the irradiance accumulated each day. Daylength or photoperiod can affect the number of leaves on maize (Bonhomme et al., 1991; Manrique and Hodges, 1991), the rate of development of sunflower (Goyne et al., 1989), and the phyllochron (the

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number of degree-days elapsed between successive leaves on a culm) in wheat and barley (Cao and Moss, 1989).

Schoolfield (1982) developed a daylength model to estimate the amount of irradiance received by soybean crops to forecast their harvest dates from satellite imagery. We have modified these equations to allow for different definitions of daylength and twilight and to enable the full range of values for latitude (CBM model). We compare daylength models to each other and to published values for daylength, show the effect of the definition of sunrise/sunset on daylength and plant growth, and discuss the significance of different definitions of daylength.

2. Daylength definitions

The length of each day varies with location upon the Earth and day of the year. Daylength also depends upon the definition one uses for the beginning and end of the day, and whether twilight is included in daylength. Some of the different definitions for sunrise and sunset are (Smithsonian Institution, 1939):

- when the center of the sun is even with the horizon,
- when the upper rim of the sun is even with the horizon,
- when the upper rim of the sun is *apparently* even with the horizon.

Refraction of light through the atmosphere causes light to illuminate a location, even when the sun is below the horizon. Refraction also causes the sun to appear to be on the horizon when it is actually below it.

If one includes twilight as part of daylength, then the different definitions of twilight must also be considered (Table 1). The light during civil twilight is considered to be bright enough to perform ordinary outdoor activities without artificial light. Civil twilight is defined as the time between sunrise or sunset and when the center of the sun is six degrees below the horizon (Smithsonian Institution, 1939). Thus, the duration of civil twilight varies with the definition of sunrise/sunset. However, if daylength is defined to include civil twilight, then the length of the day (including civil twilight) is the time from when the center of the sun is six degrees below the horizon before sunrise, until the center of the sun is six degrees below the horizon after sunset. Note that this definition of daylength starts the day before sunrise and goes until after sunset.

Table 1
Daylength definitions defined by the position of the sun with respect to the horizon

	Daylength definition (with and without twilight)	p (degrees)
1	Sunrise/Sunset is when the center of the sun is even with the horizon	0.0
2	Sunrise/Sunset is when the top of the sun is even with horizon	0.26667
3	Sunrise/Sunset is when the top of the sun is apparently even with horizon (US government definition)	0.8333 [a]
4	With civil twilight	6.0
5	With nautical twilight	12.0
6	With astronomical twilight	18.0

[a] This value is the summation of the radius of the sun (in degrees as seen from Earth) plus the adopted value for the refraction of the light through the atmosphere of 34 minutes (Astronomical Almanac 1992).

Other twilight definitions are nautical twilight, when the center of the sun is twelve degrees below the horizon, and astronomical twilight, when the center of the sun is eighteen degrees below the horizon. Again, the length of time of twilight varies with the definition of sunrise and sunset.

In the United States, the definition of daylength is usually the time between the beginning of sunrise, when the upper rim of the sun is apparently even with the horizon, until the end of sunset, when the upper rim of the sun is apparently even with the horizon (Harrison, 1960). The model presented here can simulate any of these daylength or twilight definitions. This is accomplished by selecting the number of degrees the center of the sun is at or below the horizon.

3. Model descriptions and comparisons

This section describes the CBM daylength model and models from Brock (1981), Running and Coughlan in the Forest-BCG model (1988), and Ritchie in the CERES wheat phasic model (1991).

Each model requires latitude (L) in degrees and day of year (J) as input. The output of each model is hours of light for flat, level surfaces. The definition for daylength in the Brock, BGC, and CERES models are fixed, and do not match each other's definitions in all cases nor do the definitions match the daylength definition for published values of sunrise and sunset in the Astronomical Almanac. This precludes direct comparisons between these models or to the Astronomical Almanac. The daylength definition may be specified in the CBM model. Thus, we first establish the accuracy of the CBM model via comparisons to published data, and then compare the other daylength models to the CBM model using the appropriate daylength definition in each case.

3.1. CBM model

There are several parameters which must be considered when modeling daylength. Since the beginning and ending of each day is defined by some relationship between the position of the Earth with respect to the Sun, the rotation and orbital revolution of the Earth must be modeled, and position of the flat surface on which the light is incident on the Earth must be known. Some of the mechanics which affect the accuracy of the model of the Earth's orbit are modeling an elliptical orbit rather than a circular one, and modeling the offset of the position of the Earth from the center of the circular or elliptical orbit.

To obtain the most accurate estimate for sunrise and sunset, the location on the Earth for which the daylength is to be determined and the day of the year must be selected. The problem of deriving a formula for daylength at a point on the Earth at elevation zero with non-sloping ground can be divided into three parts (Schoolfield, 1982):

1. Predicting the revolution angle (θ) from the day of the year (J).
2. Predicting the sun's declination angle (ϕ), or the angular distance at solar noon between the Sun and the equator, from the earth orbit revolution angle.
3. Predicting daylength (D) (plus twilight) from latitude (L), longitude, and the sun's declination angle.

Schoolfield's equations were modified to include the daylength coefficient (p):

$$\theta = 0.2163108 + 2\tan^{-1}[0.9671396\tan[0.00860 \times (J - 186)]], \quad (1)$$

$$\phi = \sin^{-1}[0.39795\cos\theta], \quad (2)$$

$$D = 24 - \frac{24}{\pi}\cos^{-1}\left[\frac{\sin\frac{p\pi}{180} + \sin\frac{L\pi}{180}\sin\phi}{\cos\frac{L\pi}{180}\cos\phi}\right], \quad (3)$$

p is in degrees, θ and ϕ are in radians. Also, northern latitudes are positive, while southern latitudes are negative. The arc cosine parameter of Eq. 3 (the contents of the square brackets) must fall within the range (−1.0, 1.0). The result of the sines and cosines within this parameter can lead to values greater than one and less than negative one near the poles of the Earth. In these cases, the latitude in question is either in continuous light (parameter value is greater than or equal to one) or in continuous darkness (parameter value is less than or equal to negative one).

Results from the CBM model are compared to tables for sunrise and sunset in the Astronomical Almanac 1992. The Sunrise and Sunset table in the Astronomical Almanac lists the estimated time of sunrise and sunset rounded to the closest minute for every fourth day of the year for various latitudes from 55°S to 66°N. This table uses the definition for sunrise/sunset of when the center of the sun is apparently even with the horizon (definition 3). The time of sunset was subtracted from the time of sunrise to obtain a value of daylength to within sixty seconds. The CBM model was set to use the same definition of daylength. Figs. 1a and 1b show the differences between the daylength calculated from the Sunrise and Sunset tables and the output of the CBM model rounded to the nearest minute since that is the precision of the tables.

Similar differences in modeled and extrapolated daylength from tables in the Smithsonian Meteorological Tables for the year 1899 were gathered. In this book, two tables were consulted to determine the daylength for the first day of each month. Table 95 (Declination of the Sun for the Year 1899 at Greenwich Apparent Noon) in the Smithsonian book was used to interpolate a value for the first of each month. The declination angle was then used with table 94 (Duration of Sunshine at Different Latitudes) to interpolate the value of daylength for every tenth latitude between 80° and −80°.

In these results, the latitudes ranged from 80°N to 80°S. The differences between the output of the CBM model and the published values in the Smithsonian Meteorological Tables are similar to the differences found in the Astronomical Almanac comparison. The magnitudes of the differences were slightly greater for like latitudes. Also, since the latitudes spanned a greater range, the maximum errors at the new range boundaries were greater. The largest error magnitude was −38 minutes on September 1 at 80°N followed by −31 minutes on March 1 at 80°S. The next largest errors dropped in magnitude quickly. Between 20°N and 20°S, the magnitude of error was less than or equal to 2 minutes.

These graphs show that the CBM model is fairly accurate. At latitudes near the equator, the error is 1 minute or less. Near 60°N the error is usually 2 minutes or less. The two equinoxes and solstices were chosen as dates for which the error is shown because the error was largest near these four dates (Fig. 1).

3.2. Brock model

Brock (1981) presents a method for calculating solar radiation for ecological studies where the definition for daylength is when the center of the sun is even with the horizon (definition 1).

The declination of the Earth (in degrees) as a function of day of year is calculated as:

$$\phi = 23.45 \sin\left(360 \frac{283 + J}{365}\right). \tag{4}$$

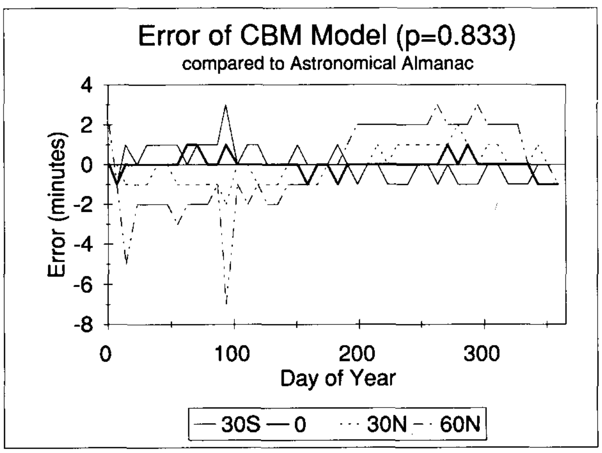
The sunset/sunrise hour-angle in degrees, with northern latitudes being positive, is calculated as:

$$hourAngle = \cos^{-1}(-\tan(L)\tan(\phi)). \tag{5}$$

Daylength in hours is then calculated as:

$$D = 2\frac{hourAngle}{15}. \tag{6}$$

Since this model uses a different definition for daylength than the Astronomical Almanac, the output from the CBM model is used for comparison. The CBM model was run with the same



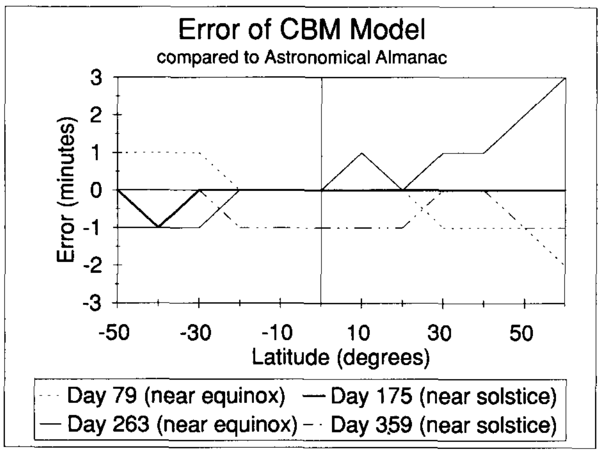


Fig. 1. Error of CBM model. Shown here are the differences, rounded to the nearest minute, between the published daylengths for year 1992 (found by subtracting the published time of sunset from the time of sunrise) and the output of CBM model for daylength for four northern latitudes (right) and four days of the year (left). The CBM model daylength definition was set to match the Astronomical Almanac ($p = 0.833$). The range of error is generally about 4 minutes, and is about 10 minutes near year day 98.

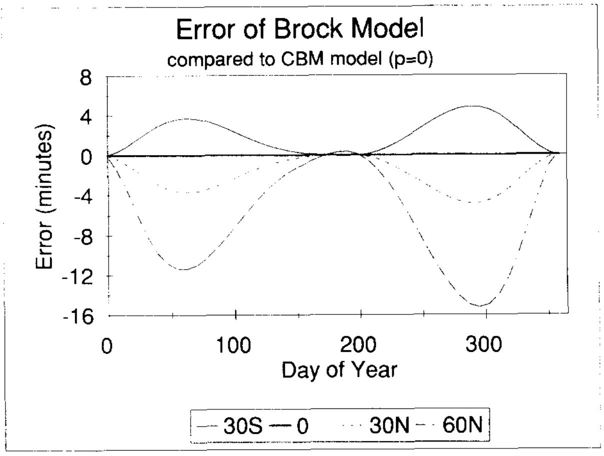


Fig. 2. Brock model comparison. The CBM model daylength definition was set to match that of the Brock model ($p = 0$). The difference at the equator, latitude zero degrees, is zero minutes. The range of error is about 20 minutes.

daylength definition as the Brock model. The difference is zero minutes at the equator and increases with latitude North and South (Fig. 2). The difference is sinusoidal with respect to the day of year with increased differences near days 55 and 295, and almost no difference near days 0 and 190.

The similarity of the daylength values produced by the CBM and Brock model at the equator, but dissimilarity at other latitudes is of interest. Further analysis of the differences in the model show that the difference in daylengths is due solely to the different equations used for solar declination in the two models. Brock uses Cooper's (1968) solar declination equation. Cooper did not state any assumptions about the equation, but said that the equation is a convenient approximate relationship. Analysis of the equation and comparison of the results of this equation to CBM's solar declination equation suggest that Cooper's equation is based upon a circular orbit of the Earth around the Sun, rather than an elliptical orbit.

3.3. BGC

The daylength calculation in the Forest-BGC model (Running and Coughlan, 1988) defines sunrise/sunset as when the center of the sun is even with the horizon (definition 1). The model calculates the amplitude of the seasonal variation in daylength from twelve hours for a given latitude, and adjusts that value depending on the day of the year.

$$ampl = e^{7.42 + 0.045L/3600}, \quad (7)$$

$$D = ampl \sin((J - 79)0.01721) + 12.0. \quad (8)$$

Running et al. (1987) define an "operational environment." They state definition 3 for daylength above, and then specify that "the daylength for net photosynthesis of a tree is more exactly defined as the period of time when the light compensation point is exceeded." This point occurs when the incoming short-wave radiation exceeds 70 W m^{-2}. Thus, the operational daylength is approximately 85% of the physical day length assuming clear skies. The result of this is similar to the use of p in the CBM model, except that Running et al. (1987) refine the results of their daylength model to adjust the output to the plant's light intensity requirements.

Examination of the BGC daylength equation shows that it is valid only for the northern hemisphere and cannot be used in the southern hemisphere. Differences between the BGC and CGM models are small at about 20° N latitude and

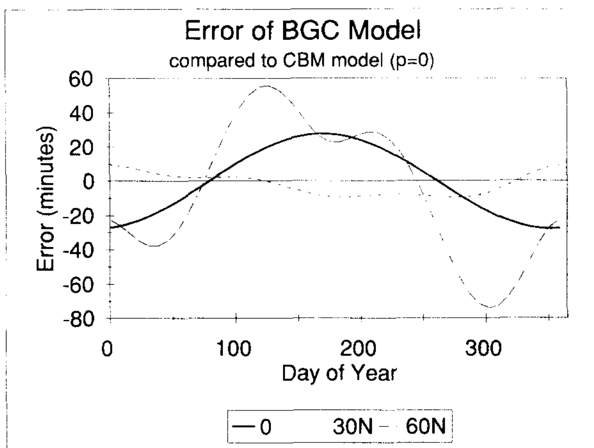


Fig. 3. BGC model comparison. The difference between the output of the BGC model and the output of the CBM model are shown for the year 1992. The CBM model daylength definition was set to match that of the BGC model ($p = 0$). The range of error is about 130 minutes at 60°N. The BGC model is not valid for the southern hemisphere.

increase with increasing latitude (Fig. 3). At 60° N, the range of error in the BGC daylength equation is about 130 minutes.

3.4. CERES model

In Ritchie's phasic development model (adapted from the CERES-Wheat model; Ritchie, 1991), the growth phases for wheat are controlled by an accumulation of daily temperature above freezing and below 26°C. The thermal time for a given day is the mean of the maximum and minimum temperatures times the number of hours of light. Thus, the plant accumulates temperature for the length of each day. Ritchie's daylength model also calculates the declination of the Earth as both the Brock and CBM models do. Ritchie's daylength model defines daylength as including the periods of civil twilight (definition 4).

Solar declination (ϕ) is in radians.

$$\phi = 0.4093 \sin(0.0172(J - 82.2)), \tag{9}$$

$$D = 7.639 \cos^{-1}\left[\max\left(-0.87, \frac{-\sin\left(\frac{L\pi}{180}\right)\sin(\phi) - 0.1047}{\cos\left(\frac{L\pi}{180}\right)\cos(\phi)}\right)\right]. \tag{10}$$

The output from this model is compared to the output of the CBM model using corresponding daylength definitions (Fig. 4). The difference is zero minutes at the equator and increases with latitude North and South, just as the Brock model does. Again, the differences between these models is mostly due to the different equations for solar declination in the two models. If the same solar declination equations are used, then there is virtually no difference in calculated daylength.

The maximization function is used to limit the range of values to the arc cosine function. The value -0.87 seems to be too high. This limitation was encountered at high latitudes during some days of the year. A value of -1.0 will increase the range of latitudes for which the CERES daylength model will provide accurate results.

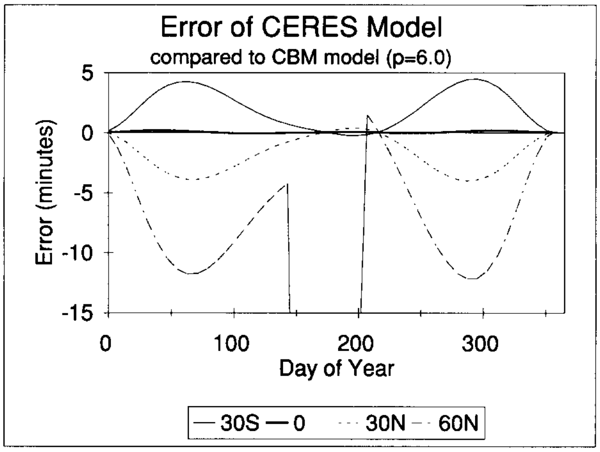


Fig. 4. CERES model comparison. The difference between the output of the Ritchie model and the output of the CBM model are shown for the year 1992. The CBM model daylength definition was set to match that of the Ritchie model ($p = 6$). The anomaly at latitude of 60°N is due to the use of a maximum function (see text). The range of error is generally about 15 minutes, and is about 170 minutes in the interval between day of year 148 and 201.

4. Cumulative effects

The growth of some plants, such as field grown winter cereals, are a function of the accumulation of hours of light (Gallagher, 1979). Some models simulate growth effects that are functions of light accumulation. As described earlier, Ritchie's phasic development model accumulates temperature for the length of each day. The definition of daylength affects the length of each day which results in different predictions of phasic development.

To illustrate the cumulative effects of the choice of daylength definition, the CERES wheat phasic model was modified to use the CBM daylength equation, and the results of different definitions are shown in Table 2. The difference in time to the termination of the spikelet phase between using civil twilight and defining sunrise/sunset as the center of the sun even with the horizon is one week.

Table 2
The effect of the definition of daylength on maturation of wheat using the CERES model. This shows the number of days calculated by the CERES model to the termination of the spikelet phase after replacing the original daylength model with the CBM model. The CERES model's original definition of daylength includes civil twilight

Daylength definition	p	Days
Sunrise/Sunset is when the center of the sun is even with the horizon	0	195
Sunrise/Sunset is when the top of the sun is even with horizon	0.26667	195
Sunrise/Sunset is when the top of the sun is apparently even with horizon	0.8333	194
With civil twilight	6.0	188

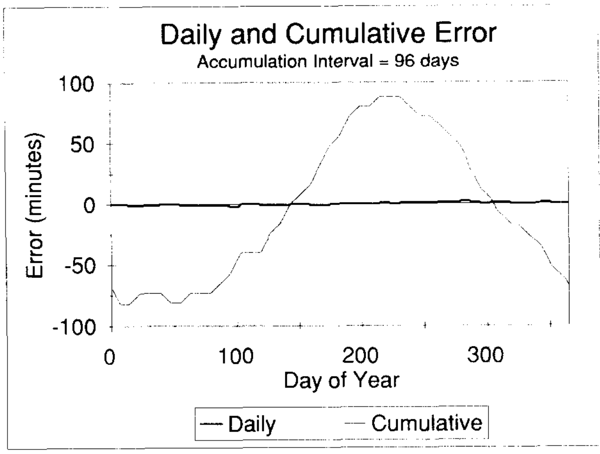


Fig. 5. Daily and cumulative error for latitude 30°N. The cumulative error line shows the accumulation of error for the next 96 days from any starting day.

If a modeler wishes to accumulate daylengths as in the CERES model, then the cumulative error may become important. To investigate the accumulation of error, the CBM model was run using daylength definition 3, and the output of the model compared to daylengths calculated from the Astronomical Almanac tables of Sunrise and Sunset. Time intervals of various lengths were chosen for which to accumulate the difference between the simulated and tabulated daylengths for each day. The starting dates for the intervals were then moved through the year to determine the maximum accumulated difference for each interval length (Table 3; Fig. 5). The lengths are from approximately one month up to six months. Longer interval lengths are not shown because the total accumulated difference decreases due to a sign change in the daily error. For intervals of approximately twelve months, the accumulated difference was close to zero. The maximum accumulated difference is −337 minutes at 60°N latitude over an interval of 160 days.

5. Error analysis

As pointed out in the previous section, the reason for the differences between the CBM model and the Brock and Ritchie models is due almost exclusively to the equation for the declination of the sun. If the same solar declination

Table 3
The maximum cumulative error in minutes for the CBM model. The interval with the largest absolute error was found for each listed latitude. Each column shows the maximum cumulative error for the interval length specified at the top of the column. In parentheses is the day of year for the beginning of the interval. See text for details

Latitude	32 days	64 days	96 days	128 days	160 days	192 days
50°S	55 (71)	98 (39)	156 (15)	203 (15)	219 (15)	219 (349)
40°S	35 (71)	67 (39)	103 (7)	133 (7)	152 (7)	144 (335)
30°S	27 (71)	54 (39)	83 (31)	104 (15)	112 (15)	119 (365)
20°S	−17 (287)	27 (15)	40 (15)	48 (15)	50 (15)	51 (15)
10°S	7 (1)	15 (1)	21 (7)	24 (1)	23 (351)	−20 (159)
0°	−12 (343)	−13 (315)	19 (15)	22 (15)	19 (359)	9 (271)
10°N	−16 (335)	−21 (319)	−30 (319)	−42 (341)	−44 (319)	−53 (319)
20°N	−24 (1)	−32 (1)	−41 (1)	−48 (341)	−56 (349)	−63 (349)
30°N	−30 (71)	−46 (39)	−76 (7)	−99 (15)	−115 (15)	−123 (365)
40°N	−39 (15)	−63 (63)	−96 (7)	−134 (7)	−150 (7)	−159 (365)
50°N	−54 (71)	−94 (39)	−146 (7)	−182 (7)	−206 (7)	−199 (365)
60°N	−80 (71)	−152 (39)	−231 (7)	−298 (15)	−337 (7)	−343 (341)

equation is used for each of these models, then the differences in the equations become the definition of daylength. For the Brock model, the daylength definition is from sunrise to sunset defined as when the center of the sun is even with the horizon. The Ritchie model's daylength definition is from the beginning of civil twilight before dawn, to the end of civil twilight after dusk. Both of these models are rigid. The CBM model, however, is flexible in that the user can provide his own definition for daylength.

This leads us to the question of which is the best equation for solar declination. The answer is based upon the modeler's judgment of accuracy required. A very complex equation set for solar declination is provided in the Astronomical Almanac. This equation set requires the calculation of the obliquity of ecliptic, ecliptic longitude, mean anomaly, Julian Date (including time of day as well as date), etc. The number of transcendental function calls and multiplications is very large. These equations provide results accurate to 0.01 degrees.

Comparison of the CBM solar declination equation to this equation set shows that a far fewer transcendental function calls and multiplications are required. The results of the CBM declination equation are within plus or minus 0.226 degrees of the Almanac equation with an average magnitude difference of 0.107 degrees throughout the year.

The CBM solar declination equation models the Earth's orbit as elliptical. If a simpler model is used (circular), then even fewer calculations are required. Brock's solar declination equation is based upon a circular orbit. Eq. 1 can be replaced by Eq. 11 to yield the circular orbit CBM solar declination equation shown as Eq. 12.

$$\phi = \frac{2\pi}{365.25}(J - 173). \tag{11}$$

The maximum difference between the Astronomical Almanac solar declination and the circular orbit CBM solar declination is 1.07 degrees with an average difference in magnitude of 0.428 degrees. The resulting differences between the CBM model output and the Astronomical Almanac tables increase by only 18%.

$$\phi = \sin^{-1}\left(0.39795 \cos\left(\frac{2\pi}{365.25}(J - 173)\right)\right). \tag{12}$$

5. Conclusion

The CBM model provides the most accurate results of the daylength models presented. The increased accuracy is due to a better model of solar declination. It is also the most robust across latitude. The error is within 1 minute near the equator, and slowly increases to 2 minutes near 60°N latitude. For all daylength models, estimates are most accurate near the equator and become less accurate at higher latitudes.

While the BGC daylength model requires relatively fewer complex mathematical operations, and will thus be faster, comparison of its output, relative to the CBM model output, suggests that the model is the least accurate of those presented here.

The Brock model is more robust than BGCs, however it approaches the complexity of the CBM model. This model, like BGCs, also uses the definitions of sunrise and sunset when the center of the sun is even with the horizon. If the CBM model is run with a sun angle of zero ($p = 0$), then the results of the Brock model and the CBM model are at times indistinguishable. In fact, at the equator, the two models' results are identical. The two models differ primarily in the equation for solar declination.

The CERES daylength model produces results which differ from the CBM model results almost identically as the Brock model results differ from the CBM model output with appropriate settings for the daylength definition. Ninety-nine percent of the difference between the Ritchie model output and the CBM model output is a result of the different solar declination equations. If all of these models use the same equation for solar declination, then the CBM model can be viewed as a transition between the Brock model and the Ritchie model, by changing the daylength definition.

Using the CBM daylength model in place of the original daylength model in Ritchie's CERES wheat phasic development model, illustrates the impact of daylength definition on models which accumulate light related values over periods of a few months. Since the impact is noticeable, the modeler must decide which definition to use. This decision is based upon the effect being modeled. To simulate onset/cessation of photosynthesis or ontogenetic phenomenon controlled by red or far red phenomenon, then the civil twilight definition should be used. In the case of plant energy balance requiring direct sunlight, then the definition one should use is where p is 0.0. In fact, if higher levels of direct irradiance are required, a negative value for p might be considered. This approach is used by Campbell (1977) in calculating energy balance.

The error analysis shows that the CBM model provides a good estimate for both daily and accumulated daylength. Substituting Eq. 11 for Eq. 1 eliminates the need for the execution of two trigonometric function calls and two multiplications and provides results with errors that are only moderately larger in magnitude.

Except for the latitudinal anomalies noted in the BGC and CERES models, all the models give estimates of daylength that are adequate for most ecological purposes. Maximum daily error ranges from 7 minutes (CBM) to about four hours (BGC) and depends on latitude. Accumulated daylength can differ by as much as seven days over a growing season. The ecological significance of such errors depends on the particular application. What is adequate for long-term dynamics such as forest growth may be entirely inadequate for short-term growth and physiological processes such as crop yields. Overall, the CBM model is the most robust of those studied, is accurate enough for any conventional ecological purpose, and is the only one that allows user selection of the daylength definition.

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